



# St. Joseph's Senior Secondary School

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## **HOLIDAY HOMEWORK-2024**

### **CLASS 11-A**

### **PHYSICS**

❖ **Make a small thin notebook for doing the given holiday homework.**

- I. Revise the content of all the chapters taught till now.
- II. Practice the solved and unsolved numericals of NCERT as discussed in class.
- III. Practice more numericals of similar type from other refresher books.
- IV. Prepare well for PA1.
- V. Attempt the following questions.
  - 1) Write in ascending order: Light year, astronomical uni, parsec.
  - 2) From the top of a building a ball is dropped while another is projected horizontally at the same time. (a) Which ball will strike the ground first? (b) Which ball will strike the ground with more speed?
  - 3) What do the slopes of distance- time and velocity -time graph represent? What do positive and negative values of these slopes imply?
  - 4) Discuss the motion of an object under free fall and draw acceleration- time velocity-time and position- time graph for this motion.
  - 5) Two vectors **A** and **B** are added. Prove that the magnitude of the resultant vector cannot be greater than  $(A + B)$  and smaller than  $(A - B)$ .
  - 6) Find the angle of projection for projectile motion whose range  $R$  is  $n$  times the maximum height  $H$ .
  - 7) Define trajectory of a projectile and hence derive equation of motion of the projectile when projected at an angle with horizontal direction.
  - 8) Establish a relation between linear velocity and angular velocity in a uniform circular motion and explain the direction of linear velocity.
  - 9) Explain angular acceleration. Establish its relation with linear acceleration.
  - 10) State parallelogram law of vector addition. find analytically the magnitude and direction of the resultant vector when (i) two vectors are parallel to each other (ii) two vectors are perpendicular to each other.

# MATHEMATICS

## Sets

1. Decide, among the following sets, which are subsets of which.  
 $A = \{x : x \text{ is a solution of } x^2 - 8x + 12 = 0\}$ ,  $B = \{2, 4, 6\}$ ,  
 $C = \{x : x \text{ is an even natural number}\}$ ,  $D = \{6\}$ .
2. State whether each of the following statements is true or false for the sets A and B where  
 $A = \{x : x \text{ is a letter in the word TRACT}\}$ ,  
 $B = \{x : x \text{ is a letter in the word CATARACT}\}$ 
  - (i)  $n(A) = 5$
  - (ii)  $n(B) = 8$
  - (iii)  $A \subset B$
  - (iv) A is a proper subset of B
  - (v)  $A = B$ .
3. Let  $\xi$  = the set of all letters in the word 'TAMILNADU' and  
 $X = \{x : x \text{ is a vowel and } x \in \xi\}$ 
  - (i) Write  $\xi$  and X in the roster form.
  - (ii) Tell  $n(\xi)$  and  $n(X)$ .
  - (iii) List all the proper subsets of X.
  - (iv) What is the cardinal number of the power set of X?
4. Let A be the set of letters in the word "POOR". Write the power set of A.
5. Find the power sets of the following sets:
  - (i)  $\{-1, 0, 1\}$
  - (ii)  $\{0, 1, \{0, 1\}\}$ .
6. If  $A = \{2, 3, 5, 7, 8\}$ ,  $B = \{1, 5, 9\}$  and  $A' = \{1, 4, 6, 9\}$ , verify that
  - (i)  $(A \cup B)' = A' \cap B'$
  - (ii)  $B - A = A' \cap B$ .
7. For all sets A, B and C, is  $A - (B - C) = (A - B) - C$  true? Justify your answer. (Exemplar)
8. If  $n(\xi) = 30$ ,  $n(A') = 15$ ,  $n(B) = 5$  and  $n(A \cap B) = 3$ , find
  - (i)  $n(A)$
  - (ii)  $n(A \cup B)$
  - (iii)  $n(A - B)$ .
9. If  $n(\xi) = 40$ ,  $n((A \cup B)') = 12$ ,  $n(A - B) = 10$  and  $n(B - A) = 14$ , find
  - (i)  $n(A)$
  - (ii)  $n(B)$
  - (iii)  $n(A \cap B)$ .
10. Two sets A and B are such that  $n(A \cup B) = 18$ ,  $n(A' \cap B) = 3$  and  $n(A \cap B') = 5$ , find  $n(A \cap B)$ .
11. Two sets A and B are such that  $n(A \cup B) = 21$ ,  $n(A' \cap B') = 9$  and  $n(A \cap B) = 7$ , find  $n((A \cap B)')$ .
12. If  $n(\xi) = 50$ ,  $n(A) = 3x$ ,  $n(B) = 2x$  and  $n(A \cap B) = x = n((A \cup B)')$ , find
  - (i) the value of x
  - (ii)  $n(A - B)$ .
13. If  $n(\xi) = 15$ , A and B are two sets such that  $A \subset B$ ,  $n(A) = 8$  and  $n(B) = 12$ , use Venn diagram to find the following:
  - (i)  $n(A')$
  - (ii)  $n(B')$
  - (iii)  $n(A \cap B')$
  - (iv)  $n(A' \cap B)$ .

# Sets

14. In a survey of 400 students in a school, 100 were listed as drinking coffee, 150 as drinking tea and 75 were listed both coffee as well as tea. Find how many students were drinking neither coffee nor tea.
  15. In an examination, 56 percent of the candidates failed in English and 48 percent failed in Science. If 18 percent failed in both English and Science, find the percentage of those who passed in both the subjects.
  16. From amongst the 6000 literate individuals of a city, 50% read newspaper A, 45% read newspaper B and 25% read neither A nor B. How many individuals read both the newspapers A as well as B?
  17. In a beauty contest, half the number of judges voted for Miss A,  $\frac{2}{3}$  of them voted for Miss B, 10 voted for both and 6 did not vote for either Miss A or Miss B. Find how many judges, in all, were present there.
  18. In a group of 50 students, the number of students studying French, English and Sanskrit were found to be as follows:  
French = 17, English = 13, Sanskrit = 15;  
French and English = 9, English and Sanskrit = 4, French and Sanskrit = 5;  
English, French and Sanskrit = 3.  
Find the number of students who study:  

|                                          |                                                         |
|------------------------------------------|---------------------------------------------------------|
| (i) French only                          | (ii) French and Sanskrit but not English                |
| (iii) English only                       | (iv) French and English but not Sanskrit                |
| (v) Sanskrit only                        | (vi) English and Sanskrit but not French                |
| (vii) atleast one of the three languages | (viii) none of the three languages. ( <i>Exemplar</i> ) |
  19. If A and B are two sets such that  $n(A) = 10$  and  $n(B) = 7$ , then find:  

|                                           |                                          |
|-------------------------------------------|------------------------------------------|
| (i) the least value of $n(A \cap B)$      | (ii) the greatest value of $n(A \cap B)$ |
| (iii) the greatest value of $n(A \cup B)$ | (iv) the least value of $n(A \cup B)$ .  |
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# Relations and Functions

- If  $f$  and  $g$  are real functions defined by  $f(x) = x^2 + 7$  and  $g(x) = 3x + 5$ , then find the values of
  - $f(3) + g(-5)$
  - $f(-2) + g(-1)$
  - $f\left(\frac{1}{2}\right) \times g(14)$
  - $f(t) - f(-2)$
  - $\frac{f(t) - f(5)}{t - 5}, t \neq 5$

(Exemplar)
- If  $f(x) = e^x$  and  $g(x) = \log x$ , then find:
  - $(f - g)(1)$
  - $(fg)(1)$
  - $\left(\frac{f}{g}\right)(3)$
- If  $f$  and  $g$  are two real valued functions defined by  $f(x) = 2x + 1$  and  $g(x) = x^2 + 1$ , then find the following functions:
  - $f + g$
  - $f - g$
  - $fg$
  - $\frac{f}{g}$

(Exemplar)
- If  $f(x) = x^3 + 1$  and  $g(x) = x + 1$  be two real functions, then find the following functions:
  - $f + g$
  - $g - f$
  - $fg$
  - $\frac{f}{g}$
  - $2g^2 - 3f$
- If  $f(x) = \sqrt{x - 2}$  and  $g(x) = \sqrt{x^2 - 1}$  be two real valued functions, then find the following functions:
  - $f + g$
  - $g - f$
  - $fg$
  - $3f - 2g$
  - $\frac{f}{g}$
  - $2f^2 + \sqrt{3}g$
  - $\frac{1}{f}$
  - $\frac{g}{f}$
- If  $f(x) = x^3 - \frac{1}{x^3}$ , prove that  $f(x) + f\left(\frac{1}{x}\right) = 0$ .
 

(Exemplar)
- If  $f(x) = x + \frac{1}{x}$ , prove that  $(f(x))^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$ .
- If  $y = f(x) = \frac{6x - 5}{5x - 6}$ , prove that  $f(y) = x, x \neq \frac{6}{5}$ .

# Relations and Functions

- If  $A = \{1, 2, 3\}$ ,  $B = \{4, 5\}$  and  $C = \{5, 6\}$ , then verify that
    - $A \times (B \cup C) = (A \times B) \cup (A \times C)$
    - $A \times (B \cap C) = (A \times B) \cap (A \times C)$
    - $A \times (B - C) = (A \times B) - (A \times C)$
  - Let  $A = \{2, 4, 6, 8\}$  and  $B = \{0, 6, 8, 9, 10\}$ . Find the elements of  $(A \cap B) \times (A - B)$  corresponding to the relation 'is a multiple of'.
  - Let  $A = \{6, 7, 8, 10\}$ ,  $B = \{2, 4, 5\}$ ,  $a \in A$ ,  $b \in B$  and  $R$  be the relation from  $A$  to  $B$  defined by  $a R b$  if and only if  $a$  is divisible by  $b$ . Write  $R$  in the roster form.
  - Let  $R = \{(x, y); x + 2y < 6, x, y \in \mathbb{N}\}$ 
    - Find the domain and the range of  $R$
    - Write  $R$  as a set of ordered pairs.
  - Let  $R = \{(x, y); y = x + 1 \text{ and } y \in \{0, 1, 2, 3, 4, 5\}\}$ .
    - List the elements of  $R$ .
    - Represent  $R$  by an arrow diagram.
  - Let  $f$  be the subset of  $\mathbb{Q} \times \mathbb{Z}$  defined by  $f = \left\{ \left( \frac{m}{n}, m \right) : m, n \in \mathbb{Z}, n \neq 0 \right\}$ . Is  $f$  a function from  $\mathbb{Q}$  to  $\mathbb{Z}$ ? Justify your answer.
- 
- Let  $f: X \rightarrow Y$  be defined by  $f(x) = x^2$  for all  $x \in X$  where  $X = \{-2, -1, 0, 1, 2, 3\}$  and  $Y = \{0, 1, 4, 7, 9, 10\}$ .  
Write the relation  $f$  in the roster form. Is  $f$  a function?
  - Is  $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$  a function? If this is described by the relation  $g(x) = \alpha x + \beta$ , then what values should be assigned to  $\alpha$  and  $\beta$ . (Exemplar)
  - Determine a quadratic function ' $f$ ' defined by  
 $f(x) = ax^2 + bx + c$  if  $f(0) = 6$ ,  $f(2) = 11$  and  $f(-3) = 6$ .
  - Find the domain and the range of the function  $f(x) = 2 - 3x^2$ . Also find  $f(-2)$  and the numbers which are associated with the number  $-25$  in its range.
  - Find the domain and the range of the following functions:
    - $\sqrt{x-3}$
    - $\sqrt{25-x^2}$
    - $5 - |x+1|$ .
  - Draw the graph of the function  $f(x) = \begin{cases} 1+2x, & x < 0 \\ 3+5x, & x \geq 0 \end{cases}$ .  
Hence, find its range.
  - If  $f(x) = 2x + 5$  and  $g(x) = x^2 - 1$  are two real valued functions, find the following functions:
    - $f + g$
    - $f - g$
    - $fg$
    - $\frac{f}{g}$
    - $\frac{g}{f}$
    - $3g + 2f^2$ .

# Trigonometry

- Find the value of  $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ .
- If  $\tan \theta = 3$  and  $\theta$  lies in the third quadrant, then find the value of  $\sin \theta$ .
- Write  $\tan\left(\frac{3\pi}{2} + \theta\right)$  in terms of  $\theta$ .
- Range of secant function is  $R - (-1, 1)$ . State true or false.
- Find the value of  $\sin\left(\frac{-11\pi}{3}\right)$ .
- Find the value of  $\cot\left(\frac{-15\pi}{4}\right)$ .
- Find the value of  $\cos 210^\circ$ .
- Find the value of  $\tan(-1125^\circ)$ . [DoE]

## Section A

- Find the value of  $\sin \frac{5\pi}{3}$ .
- Prove that  $\tan 225^\circ \cdot \cot 405^\circ + \tan 765^\circ \cdot \cot 675^\circ = 0$ .
- Prove that  $2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$
- Simplify 
$$\frac{\tan(90^\circ - \theta) \sec(180^\circ - \theta) \sin(-\theta)}{\sin(180^\circ + \theta) \cot(360^\circ - \theta) \operatorname{cosec}(90^\circ - \theta)}$$
- Find  $x$  from the equation:  $\operatorname{cosec}(90^\circ + A) + x \cos A \cot(90^\circ + A) = \sin(90^\circ + A)$ .
- Prove that  $\frac{\sin 225^\circ - \cos 120^\circ}{\sin 225^\circ + \cos 120^\circ} = (3 - 2\sqrt{2})$ .

# Trigonometry

## Section B

- Find the value of  $\tan 75^\circ - \cot 75^\circ$ .
- Find the minimum value of  $3 \cos x + 4 \sin x + 8$ .
- Simplify:  $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$
- Prove that  $\frac{\sin 7\alpha - \sin \alpha}{\sin 8\alpha - \sin 2\alpha} = \cos 4\alpha \cdot \sec 5\alpha$ .
- Find the value of  $\sin 32^\circ \cos 28^\circ + \cos 32^\circ \sin 28^\circ$ .
- Express as sum or difference,  $2 \cos 4\theta \cos 6\theta$ .
- Express  $\sin 5\theta - \sin 7\theta$  as a product.
- Evaluate,  $\sin 105^\circ + \cos 105^\circ$ .
- Prove that  $\frac{\sin \alpha - \sin \beta}{\cos \beta - \cos \alpha} = \cot \frac{\alpha + \beta}{2}$ .
- If  $\alpha, \beta$  are the solutions of the equation  $a \cos \theta + b \sin \theta = c$ , then show that 
$$\cos(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2}$$

# Trigonometry

11. Prove that,  $\tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$ .
12. Prove that  $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$
13. Find  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$ , if  $\tan x = -\frac{4}{3}$ , where  $x$  lies in 2nd quadrant.
14. Prove that  $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$
15. Prove that,  $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$  [NCT 2011]
16. Prove that  $\frac{\sin 5x - 2 \sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$  [NCT 2013]
17. Prove that  $\tan 13A - \tan 7A - \tan 6A = \tan 13A \tan 7A \tan 6A$
18. If  $A + B = 45^\circ$ , then prove that  $(1 + \tan A)(1 + \tan B) = 2$  [DoE]
19. Prove that  $\frac{2 \sin(\alpha - \gamma) \cos \gamma - \sin(\alpha - 2\gamma)}{2 \sin(\beta - \gamma) \cos \gamma - \sin(\beta - 2\gamma)} = \frac{\sin \alpha}{\sin \beta}$
20. Prove that  $\cos \alpha \cdot \cos(60^\circ - \alpha) \cos(60^\circ + \alpha) = \frac{1}{4} \cos 3\alpha$
21. If  $\cos(\theta + 2\alpha) = m \cos \theta$ , prove that  $(1 - m) \cot \alpha = (1 + m) \tan(\theta + \alpha)$ .
22. Prove that  $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$  [DoE]
23. Prove that  $\cos^2 A + \cos^2(A + 120^\circ) + \cos^2(A - 120^\circ) = \frac{3}{2}$ . [NCT 2019]
24. Prove that  $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$  [NCERT]
25. Prove that  $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$  [NCERT]
26. If  $\alpha, \beta$  are two distinct roots of the equation  $a \tan \theta + b \sec \theta = c$ , prove that  $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$  [DoE]
27. If  $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$ , then prove that  $\cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$ . [NCERT Exemplar]
28. Evaluate  $\cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{5\pi}{7} + \cos \frac{6\pi}{7} + \cos \frac{7\pi}{7}$  [NCT 2016]

## Section C

1. Find the value of  $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ .
2. If  $\sin \theta + \cos \theta = 1$ , then find the value of  $\sin 2\theta$ .
3. If  $\alpha + \beta = \frac{\pi}{4}$ , then find the value of  $(1 + \tan \alpha)(1 + \tan \beta)$ .
4. Evaluate  $\sin \frac{7\pi}{12} \cos \frac{\pi}{4} - \cos \frac{7\pi}{12} \sin \frac{\pi}{4}$ .
5. Write  $\frac{13\pi}{4}$  in the degrees.
6. Express  $-47^\circ 30'$  in radian measure.
7. What is the value of  $\sin \frac{31\pi}{3}$ ? [NCT 2011]
8. Express the angle in sexagesimal system (i.e. in degrees, minutes, seconds)  $\frac{\pi^c}{12}$ .
9. Express the given angle in radians  $5^\circ 37' 30''$ . [DoE]
10. A train is travelling on a curve of 700 m radius at 14 km/h, through what angle will it turn in one minute?
11. Find the value of  $\sin 315^\circ$ .
12. Find the value of  $\tan \frac{11\pi}{6}$ .
13. Find the value of  $\cot 225^\circ$ .
14. In triangle ABC, prove that  $\sin(A + B) = \sin C$ .
15. In triangle ABC, prove that  $\tan\left(\frac{B + C}{2}\right) = \cot \frac{A}{2}$ .

# Trigonometry

16. In quadrilateral ABCD, prove that  $\cos(A + B) = \cos(C + D)$ .
17. In cyclic quadrilateral ABCD, prove that  $\sin A = \sin C$ .
18. Find the value of  $\sin 38^\circ 30' \cdot \cos 21^\circ 30' + \cos 38^\circ 30' \cdot \sin 21^\circ 30'$ .
19. Find the value of  $\cos 47^\circ \cdot \sin 17^\circ - \sin 47^\circ \cdot \cos 17^\circ$ .
20. Find the value of  $\cos 85^\circ \cdot \cos 40^\circ + \sin 40^\circ \cdot \sin 85^\circ$ .
21. Express as sum or difference  $\cos 3\theta \sin \theta$ .
22. Express  $\cos \theta - \cos \frac{7\theta}{2}$  as a product.
23. Prove that  $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$ .
24. Prove that  $2 \sin\left(\frac{\pi}{4} + \alpha\right) \cos\left(\frac{\pi}{4} + \beta\right) = \cos(\alpha + \beta) + \sin(\alpha - \beta)$
25. Prove that  $\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$
26. What is the maximum value of  $3 - 7 \cos 5x$ ? [DoE]
27. The difference between two acute angles of a right-angled triangle is  $\frac{3\pi}{10}$  radians. Express the angles in degrees.
28. Prove the identity  $\frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ} = \tan 36^\circ$ .
29. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord. [NCERT, HOTS]
30. Prove that  $\cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$ . [NCERT, HOTS]
31. Prove that  $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$ . [NCERT]
32. Prove that  $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$ . [NCERT]
33. Find the minimum and maximum value of  $\sin^4 x + \cos^4 x$ ,  $x \in \mathbb{R}$ . [DoE]
34. If  $\tan A - \tan B = x$ ,  $\cot B - \cot A = y$ , prove that  $\cot(A - B) = \frac{1}{x} + \frac{1}{y}$ . [DoE]
35. If A and B are acute angles such that  $\tan A = \frac{p}{p+1}$  and  $\tan B = \frac{1}{2p+1}$ , show that:  
(i)  $A + B = \frac{\pi}{4}$ .  
(ii)  $\tan A + \tan B + \tan A \tan B = 1$ .
36. Prove that  $\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}$ . [DoE, NCT 2014]
37. Solve for x,  $4 \sin x \sin 2x \sin 4x = \sin 3x$ . [DoE]
38. Evaluate  $\left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right)$ . [DoE]
39. Find the angle in radians through which a pendulum swings if its length is 75 cm and the tip describes an arc of length 21 cm.
40. Prove that,  $\frac{\tan(A + B)}{\cot(A - B)} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$ .
41. If  $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$ , prove that  $xy + yz + zx = 0$ . [NCT 2019, 2017, 2016]
42. Prove that  $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$ .

# Trigonometry

26. Prove that:

$$(i) \frac{1 - \cos x + \cos y - \cos(x+y)}{1 + \cos x - \cos y - \cos(x+y)} = \tan \frac{x}{2} \cot \frac{y}{2}$$

$$(ii) \frac{\cos^3 x - \cos 3x}{\cos x} + \frac{\sin^3 x + \sin 3x}{\sin x} = 3.$$

27. Prove that  $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$

28. Show that  $\frac{\tan 3x}{\tan x}$  never lies between  $\frac{1}{3}$  and 3.

☆ Do Holiday HW in a separate notebook.

29. Solve the following equations:

$$(i) \tan 2x = -\cot \left( x + \frac{\pi}{6} \right)$$

$$(ii) \cot^2 x + 3 \operatorname{cosec} x + 3 = 0$$

$$(iii) 4 \sin^2 x + \sqrt{3} = 2(1 + \sqrt{3}) \sin x$$

$$(iv) \tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 0$$

$$(v) \cos 2x - \cos 8x + \cos 6x = 1$$

$$(vi) \tan \left( \frac{\pi}{4} + x \right) + \tan \left( \frac{\pi}{4} - x \right) = 4$$

$$(vii) \operatorname{cosec} x = 1 + \cot x.$$